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**Project 6 – Numeric Computations with Taylor Polynomials**

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Course number: CST - 305

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**Project 6 – Numeric Computations with Taylor Polynomials**

**Description**

**PART 1: Taylor Polynomial Series Method**

[ Equation 1 – a ]

We have a given conditions with a given equation. Part 1-a is to construct the Taylor expansions of f(x) manually. In this documentation, we will find the value of y at the point 3.5 and calculate the n value up to 4. Finally, Python program will visualize the Taylor series and its convergence.

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First, we have to define the up to ,

If we solve the differential equation with our initial values when , then we get,

Since we have found the value of when , we have to differentiate the second order derivative to find third and fourth level of derivatives.

Apply found values of term of series.

Now we have to find to express terms of Taylor expression.

Apply found values of term of series.

Now we have found the 4th n-terms. Now we have to apply into the Taylor Series function to solve differential equation.

The function of the form is given as below:

If we apply the function to the 4th term, then we get:

Now we have a solution for , then apply 3.5 to find the value of y.

[ Visualization of Part 1-a]

Chart

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[ Equation 1 – b ]

Second part of the Part 1 is finding the second order Taylor polynomial near x = 3. With the given equation and initial value problem, we have to find the Taylor polynomial equation and visualize it.

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Since we have given a near x point, so we can find a value of near by solving the differential equation with the initial value problem.

Move everything besides and solve the equation by substitute the values with IV.

Now, since we have a value for , we can find the solution for the differential equation by using the Taylor expansion formula.

Now, we have to find a value of near x = 3 to solve the problem.

.045

As x approaches 3, the solution value is pointing near the integer 6.

[ Visualization of Part 1-b ]

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**PART 2: Power Series Method**

Part 2 is to determine whether the given value of x is an ordinary point of the given differential equation. If x is an ordinary point, find the recurrence formula up to and then the general solution of the given differential equation. The given equation is as below:

[ Evaluate an Ordinary Point ]

By using the differential ordinary equation, we can define the value of x is an ordinary point or not.

By checking and , we can define the ordinary point.

, since is 0

, and when x = 0, .

Since both and are constant, x is an ordinary point of the differential equation.

[ Find Recurrence Formula ]

We can use Power series method to find the recurrence formula. A recurrence formula (also known as a recurrence relation) is a mathematical equation that defines a sequence of numbers based on one or more of its previous terms. It expresses the nth term of the sequence in terms of one or more of the preceding terms. So generally, we have to find the formula that corresponds to the solution at n-terms.

First, we have to define each individual series of terms.

Second, substitute each individual sequence term to the equation that is given.

Calculate the equation above and ignore the other term besides , then we get:

Even though the equation given is nonhomogeneous function, if we expand the equation to the numerous terms, only x value exists in the expansion. Therefore, the right side becomes 0.

Solve for to find the recurrence formula:

Find the recurrence sequence for :

Since we found the recurrence sequence for 8th term, then the general solution in an interval containing this point has the form as this:

We can expand this form into this:

Now, since we found the values up to n = 8, substitute each individual values to get a general solution

Simplify it into form, then we get:

We have found the and , so by using one of the second order differential equation method, we can find the and which are constant values.

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**PART 3: The Computer Performance Analysis**

The model we will use is the performance of the computer network bandwidth. The network bandwidth is a measure of the system performance that indicates the amount of data that can be transmitted over a network in each period. It is calculated by taking the rate of change of the network bandwidth, and the amount of data transmitted over the network. A high network bandwidth is desirable for a system as it ensures that data can be transmitted quickly and efficiently. However, if the network bandwidth is too low, it can lead to delays and bottlenecks in the system, resulting in a poor user experience. Therefore, it if the amount of data transmitted over a time states the how fast the data flows in between the system.

1. The equation given to calculate ODE is dy/dt = rate – y

* rate: represents the rate of change of the network bandwidth.
* y: represents the amount of data transmitted over the network.
* dy/dt: rate of change at which the amount of data is transmitted over the network.

1. This ODE finds the rate at which the amount of data is transmitted over the network, therefore it is important to know the relationship between the amount of data transmitted over the network and the rate of change of the network bandwidth.

In conclusion, this assignment required the use of numerical methods to solve a differential equation that models computer performance. The assignment was divided into three parts, which involved constructing a Taylor expansion of a function with multiple variables, using the Taylor expansion to calculate a numeric solution to a nonlinear differential equation, and modeling a problem in the context of a computer system. In the first part, students were required to solve a differential equation and manually construct the Taylor expansion of a function. The second part involved determining whether a given value of x is an ordinary point of a differential equation and finding the recurrence formula and general solution if it is. In the final part proposed a naïve, holistic model for assessing computer performance and calculated the performance of the system that we have done in topic 1.